Monte Carlo Simulations of Freeze-out with Momentum Constraints in High Energy Nuclear Collisions

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Abstract

Existing Monte Carlo freeze-out algorithms usually do not account consistently for all conservation laws, such as the conservation of momentum or the conservation of energy. This poster will document the author's work during his REU program, to develop a Monte Carlo simulation modeling the freeze-out from high energy nuclear collisions imposing momentum conservation. We briefly explain the sampling algorithm that exactly enforces momentum conservation for non-relativistic Maxwell-Boltzmann distributions and test how it performs as an approximation to relativistic Bose-Einstein and Fermi-Dirac systems. Once completed, our simulation could provide important insights into the behavior of quark gluon plasma and high energy collisions.

Freeze-out

When nuclei collide with high energy, the fireball created in the collision expands and cools rapidly so that the particles stop interacting with each other. This is known as freeze-out. Freeze-out happens to the hadrons created from quark gluon plasma. Understanding freeze-out is important for understanding high energy collisions, and quark gluon plasma, which is a primordial state of matter.

Monte Carlo (MC) Sampling

Statistical physics uses average particle distributions to describe systems. The most important ones are distributions in thermal equilibrium,

$$f(p,T) = \frac{1}{e^{\frac{E}{T}} - S}$$

where p is the momentum, T is the temperature, E is the energy of a particle with momentum p, and s=0, 1, -1 depending on whether you are using the Maxwell-Boltzmann, Bose-Einstein, or Fermi-Dirac distributions respectively. Monte Carlo sampling has something of a reverse task, it generates an ensemble of individual particles that match their statistical distribution function.

Basic MC Algorithm

To sample a distribution for $f(\vec{p})$, a table was first generated for the integral of the distribution function, i.e. the probability of a particle having a momentum up to that magnitude. Then, a random number generator was used to choose a momentum from the table. The direction of the particle was randomly chosen. By applying this algorithm several times, an ensemble of particles can be obtained which represents this distribution function.

In the case of the fireball created in nuclear collisions, in addition to the thermal motion that the distribution function accounts for, there is also a collective motion outward at relativistic speeds. To account for this, we generate particles in the local rest frame, and then use the Lorentz transformation to determine the particle momenta relative to the lab frame.

Constraints

Often a system will have more constraints than just their single particle distribution. For example, the system could have a known net momentum, energy, electric charge, etc. A Monte Carlo sampling based solely on single particle distribution will typically violate these constraints and have an error of order O($\sqrt{N}$) where N is the number of particles in the system. Constraints lead to non-trivial correlations between particles, which are often not explicitly known. Existing algorithms apply a posteriori selections to enforce constraints, leading to questionable consistency.

Algorithm with Momentum Constraint

An algorithm to enforce a given total momentum exactly for a non-relativistic Boltzmann system is known [1]. Here we assume $\overrightarrow{P_{tot}} = \vec{0}$, other cases are related by a simple Lorentz boost. In this procedure, every time we generate a particle we use a unique effective temperature and momentum offset in the cumulative distribution function. For a system with N particles, the temperature for the ith particle is

$$T_i = T * \frac{N*(N-(i-1)-1)}{(N-1)*(N-(i-1))}$$

and the offset (momentum kick) is

$$\overrightarrow{p_i^{offset}} = -\overrightarrow{p_{i-1}^{net}} * \frac{1}{N-(i-1)}$$

where

$$\overrightarrow{p_{i-1}^{net}} = \sum_{j=1}^{i-1} \overrightarrow{p_j}$$

is the net momentum of the previously sampled particles. The final particle has a momentum equal and opposite to the sum of all previous momenta. In this way we can easily demonstrate the conservation of momentum. We are exploring how well this algorithm works when applied as an approximation to freeze-out in high energy nuclear collisions.

Results I: Static Box

We apply the previous algorithm to a box with relativistic bosons at constant temperature T. The algorithm naturally enforces $\overrightarrow{p_{tot}} = \vec{0}$. In addition, we find that the single particle distribution matches quite well the analytic Bose-Einstein distribution at the same temperature.

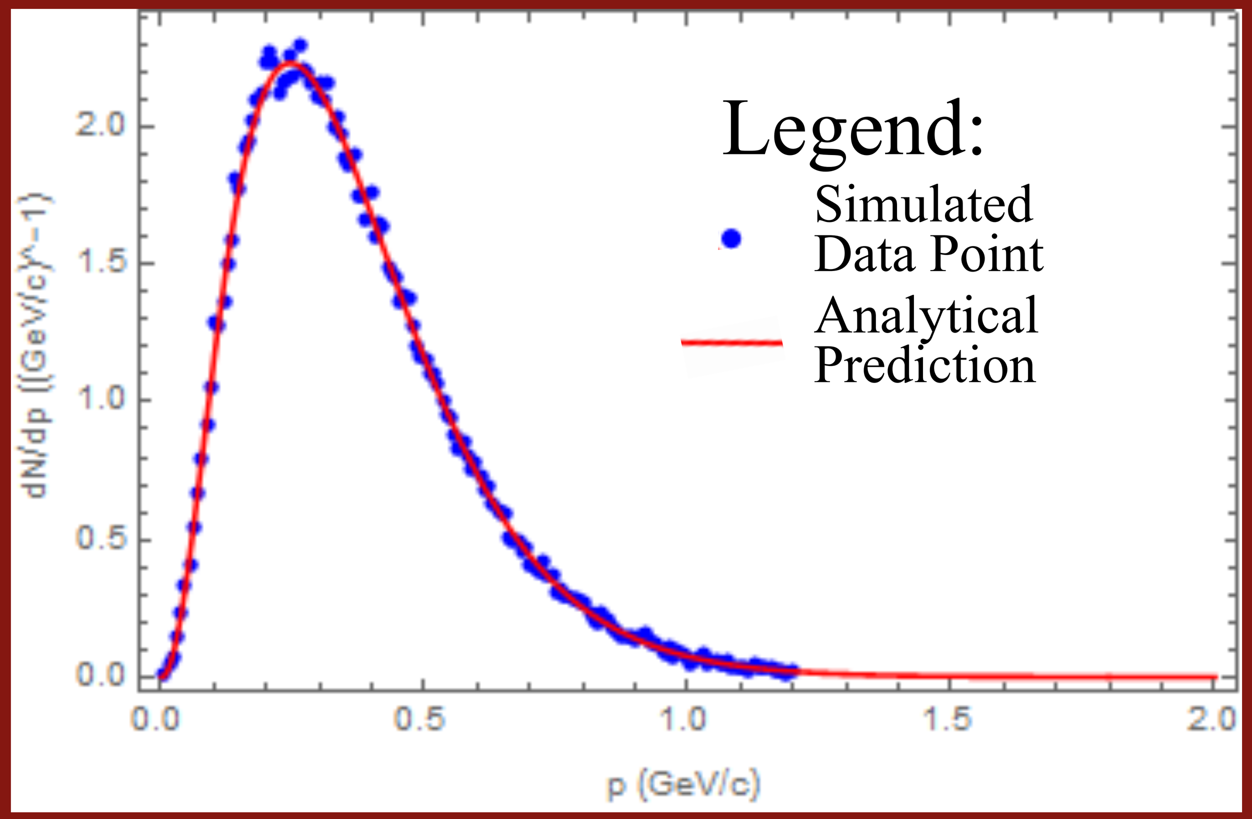

Figure 1: Momenta distribution in a single uniform cell. T = 120 MeV, m = 130 MeV, N = 100,000.

Results II: Freeze-Out in Nuclear Collisions

We now apply the algorithm to a realistic freeze-out hypersurface and collective flow field in nuclear collisions. In our case, surface and velocity field have been calculated in a relativistic fluid dynamic simulation [2].

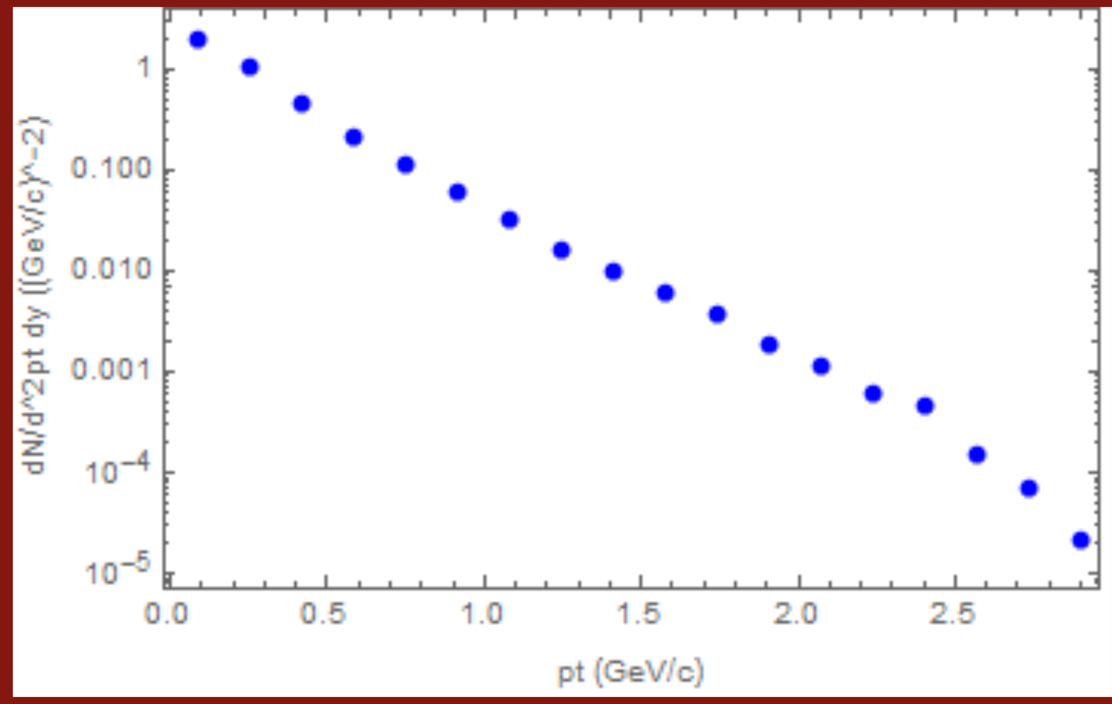

Figure 2: Log plot of transverse momentum distribution of pions in a fireball with hypersurface and flow field similar to a central Pb+Pb collision at LHC, T=120 MeV.

Summary

We have tested an algorithm that enforces total momentum conservation for the sampling of classical, non-relativistic particle systems and find that it numerically leads to good results also for quantum (Bose, Fermion) systems with relativistic motion. Important applications in high energy nuclear collisions await. As a next step we will implement and test an algorithm to enforce a total energy conservation.

Citations

[1] D. Molnar, Private Communication
[2] S. Somanathan, Private Communication